Machine Learning – Basics of Linear Algebra

**This is a recap + tantecose di linear algebra, some things are considered known**

**Norms**

The norm of a vector is its length, with the most common norm beng the l2 norm (or **Euclidean norm**):

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Description automatically generated with low confidenceA norm is a function f : R n → R must satisfy:

* Non-negativity for all x, f(x) >= 0
* Definiteness f(x) = 0 iff x = 0
* Closure wrt to product by a scalar i.e. **f(tx) = |t|f(x), t ∈ R**
* Closure wrt sum i.e. **f(x+y) = f(x) + f(y)**

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**Orthogonal Matrices**

The orthogonality of vectors is defined by their dot product:

<v,w> = 0 iff they are orthogonal

**A matrix is orthogonal if all of its columns are pairwise orthogonal and normalized** (orthonormal)

It follows that:

And that the transpose is equal to the inverse

**Affine spaces and sets**

The affine space exists independently from the chosen basis, as it lacks the usual coordinate system (i.e., the origin)

There is instead the notion of **translation vectors** between points in the space

A set is affine **iff** it contains all lines through any two points in the set (the set contains the linear combination of any two points in it, provided that all the coefficients sum to 1)

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Each vector space can be regarded as an affine space A(V, V, α) where α(x,v) is the sum of x and v in V

The affine subspaces of A(V) are the sets x+W={x+w∣w∈W} where:

* x∈V
* W is linear subspace of V

**Hyperplanes**

A hyperplane is an affine set (called space by prof) of dimension n-1, it generalizes the notion of common plane

It divides the space into two half-spaces

Diagram

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**Projection onto a Subspace**

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